Comparative Studies of Atmospheric Density Models Used for Earth Satellite Orbit Estimation

R.E. Shanklin Jr.,* T. Lee,† M. Samii,‡ M.K. Mallick,‡ and J.O. Cappellari Jr.§
Computer Sciences Corporation, Silver Spring, Maryland

The results of a comparative orbit determination study of four global atmospheric density models (modified Harris-Priester, Jacchia-Roberts, Mass Spectrometer/Incoherent Scatter, and Simple Exponential Model) are presented. The primary objective of this study was quantitative evaluation of the definitive orbit determination consistency and accuracy obtained using these models as measured by the maximum position differences that occur during solution overlap periods. A secondary objective was comparison of propagated ephemerides with definitive orbit solutions in order to evaluate predictive accuracies. The results indicate that, for satellites above 300 km, all four atmospheric density models produce comparable orbit determination accuracies when an atmospheric drag scaling factor and the satellite state vector are estimated during the orbit determination process.

I. Introduction

ATMOSPHERIC drag causes a significant perturbation of Earth satellite orbits with perigee heights of less than 1000 km. The acceleration of a spherical satellite due to atmospheric drag is given by the equation

$$a_{\text{drag}} = -\tfrac{1}{2}(C_D/m)A\rho\,|V|\,V$$

where ρ is the atmospheric density at the position of the satellite, V the satellite velocity relative to the atmosphere, A the satellite reference cross-sectional area, C_D the satellite drag coefficient, and m the satellite mass. Therefore calculation of the drag acceleration requires knowledge of the atmospheric density as a function of position and time.

This paper presents the results of a comparative study of four global atmospheric density models in the context of orbit determination. The models compared are the Harris-Priester (HP) model, the Jacchia-Roberts (JR) model, the Mass Spectrometer/Incoherent Scatter (MSIS) model, and a Simple Exponential Model (SEM).

The Harris-Priester model is based on theoretical temperature profile solutions of the heat conduction equation under hydrostatic equilibrium conditions. The diurnal variation is modeled by a correction calculated using a power of a cosine.[1,2]

The Jacchia-Roberts model is based on empirical temperature profiles scaled by an upper boundary exospheric temperature (T_∞). Analytic density calculation is accomplished through integration of thermodynamic equations. The modeling includes corrections for extreme ultraviolet (EUV) heating, solar particle flux (so-called geomagnetic) heating, semiannual variations, seasonal variations, and the diurnal variation.[2,3]

The MSIS model[4] is based on fitting spherical surface harmonic expansions to match the angular dependence exhibited by mass spectrometer and incoherent scatter measurements. The MSIS formulation includes sections that model EUV heating, solar particle flux heating, annual variations, semiannual variations, diurnal variations, semidiurnal variations, terdiurnal variations, and departures from diffusive equilibrium.

The SEM calculates unperturbed atmospheric densities using the expression

$$\rho = \rho_0 e^{-SH}$$

where H is the altitude, and ρ_0 and S are constants. The accuracy of orbit determination results obtained using this model is insensitive to the selection of ρ_0 because of the d_l estimation capability (see Sec. II). The value of the scale height parameter (S) depends upon the EUV heating level and governs the slope of the density profile. Thus, for orbits of relatively high eccentricity (e.g., Magsat), S must be chosen carefully. The simple model also includes a diurnal bulge variation, and thus the density is given by

$$\rho' = \rho_0 e^{-SH}\,[1+\cos^4(\phi/2)]$$

where ϕ is the geocentric angle between the satellite and the apex of the diurnal bulge.

The relative density profiles of the four atmospheric models are illustrated in Fig. 1.

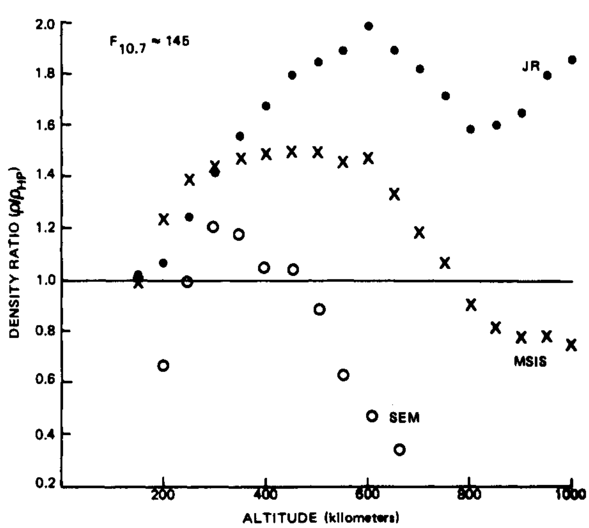

Fig. 1 Atmospheric model density ratios.

Presented as Paper 82-1445 at the AIAA/AAS Astrodynamics Conference, San Diego, Calif., Aug. 9-11, 1982; received Aug. 17, 1982; revision received March 21, 1983. Copyright © 1982 by R.E. Shanklin Jr. et al. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission.

*Staff Scientist, System Sciences Divison. Member AIAA.
†Staff Scientist, System Sciences Division.
‡Member Technical Staff, System Sciences Division.
§Senior Scientist, System Sciences Division. Member AIAA.

II. Comparative Study Structure

All the results presented in Sec. III of this paper are based on Goddard Trajectory Determination System (GTDS) Bayesian weighted least-squares differential correction solutions. Three different low-altitude satellites (AE-3, Magsat, and SAGE) were studied; orbital parameters for these satellites are given in Table 1. Nine different series (three for AE-3, five for Magsat, and one for SAGE) of six GTDS Differential Correction (DC) Program runs were made for each of the four atmospheric models. Each series contained six 30-hour-arc solutions.

Each differential correction solution consisted of seven numbers: three position coordinates, three velocity coordinates, and the drag variation parameter (d_l), which is a scaling factor in the drag acceleration equation, i.e.,

$$a_{\text{drag}} = -\tfrac{1}{2}(C_D/m)A\rho(1+d_l)\,|V|\,V$$

This scaling factor was applied during the generation of each ephemeris.

Spacecraft attitude was not considered, since a spherical model was employed. Furthermore, no aerodynamic forces (e.g., lift) other than drag were modeled. However, it is reasonable to expect that both of these assumptions had a negligible effect on the results of this study, because the results were obtained by applying each of the four atmospheric models to the same arcs with the same observation sets. Simply stated, unmodeled aerodynamic forces should have perturbed the solutions for all four atmospheric models in a similar manner.

In order to evaluate definitive orbit determination accuracy, the differential correction solutions were used to generate 30-hour ephemerides that overlapped adjacent ephemerides by 6 h. These ephemerides were then compared in order to determine the maximum position differences during the overlap periods. The 216 DC Program solutions produced 180 maximum overlap position differences. These differences were used to evaluate the consistency and accuracy obtained when each of the four atmospheric density models was used.

Table 1 Satellite orbital elements

Satellite	Date	Perigee height, km	Apogee height, km	Inclination, deg
AE-3	August 1, 1978	331	341	68
Magsat	October 31, 1979	352	561	97
Magsat	March 1, 1980	323	471	97
SAGE	February 19, 1979	560	655	55

Table 2 Comparison of results obtained using the four atmospheric models

	Harris-Priester model		Jacchia-Roberts model	
Spacecraft	Average weighted rms*	Average maximum position difference, m	Average weighted rms	Average maximum position difference, m
AE-3	7.3	225	7.8	217
SAGE	9.9	203	10.2	210
Magsat	9.4	213	9.5	166
	MSIS model		Simple exponential model	
Spacecraft	Average weighted rms*	Average maximum position difference, m	Average weighted rms*	Average maximum position difference, m
AE-3	8.5	324	7.3	174
SAGE	10.0	213	10.0	216
Magsat	11.3	288	9.1	246

*The root mean square (rms) is presented for comparison purposes.

Table 3 Predictive study position differences

	Average maximum position difference, km					
	Harris-Priester model		Jacchia-Roberts model		Simple exponential model	
Spacecraft	During 2nd day	During 5th day	During 2nd day	During 5th day	During 2nd day	During 5th day
AE-3	5.4	29.3	3.4	30.4	4.2	27.8
Magsat	1.5	4.6	4.5	8.8	1.2	7.6

In order to evaluate predictive orbit determination accuracy, three 6-day (1-day definitive, 5-day predictive) ephemerides were generated using the differential correction solutions from the first arcs of two AE-3 series and one Magsat series. These predictive ephemerides were then compared with the definitive ephemerides for the third and sixth arcs of the three series, thereby determining the maximum position differences for the second and fifth predictive days.

III. Definitive Study Results

This section presents the results of this comparative study in the context of definitive orbit determination. Table 2 summarizes these results.

For the three AE-3 series, the average maximum position difference during the 6-hour overlap periods for the Jacchia-Roberts model was about 4% (8 m) smaller than the Harris-Priester average; the MSIS average was about 44% (99 m) larger than the Harris-Priester average; and the SEM average was about 23% (51 m) smaller than the Harris-Priester average. However, even the 107-m difference between the Jacchia-Roberts and MSIS averages cannot be considered to be either large or significant.

The results for the five Magsat series show that the Jacchia-Roberts average maximum position difference was about 22% smaller than the Harris-Priester average difference and that the MSIS and SEM average differences were 35% and 15% larger, respectively, than the Harris-Priester average difference. As in the case of AE-3, the Magsat results demonstrate that the atmospheric density models are comparable in the context of this study.

The average maximum overlap position differences for the SAGE series agreed to within 7%; thus all four atmospheric models produced essentially equivalent errors for SAGE.

IV. Predictive Study Results

This section presents the results of a study of comparative predictive accuracy for the Harris-Priester, Jacchia-Roberts, and SEM atmospheric density models. Two AE-3 and one Magsat series were evaluated. In this evaluation, a predictive ephemeris generated using the solution for the first arc (day) in each series was compared with definitive ephemerides generated for the third and sixth arcs (days) of that series, yielding a measure of the error in the prediction during the second and fifth predictive days. Table 3 summarizes the results of these comparisons.

The average maximum position difference for the two AE-3 series using the Jacchia-Roberts model is 19% (0.8 km) smaller than the SEM average maximum difference and 37% (2.0 km) smaller than the Harris-Priester average maximum difference after a 2-day predictive time span.

After a 5-day span, however, the Jacchia-Roberts average maximum position difference was 9% (2.6 km) and 4% (1.1 km) larger than the SEM and Harris-Priester average differences, respectively. The Magsat results show that after 2 days of prediction, the Jacchia-Roberts maximum position difference was about three times larger than either the Harris-Priester or the SEM maximum position difference (which differ by only 20% from each other). After 5 days of prediction, the SEM maximum position difference was 65% larger than the Harris-Priester maximum position difference, and the Jacchia-Roberts maximum position difference was about twice as large as the Harris-Priester maximum position difference.

From these results, it is clear that the accuracy with which the atmospheric density models produce predictive ephemerides varies with the orbital parameters of the spacecraft and the time period of the study. Moreover, none of the models consistently produces the lowest comparison results, which leads to the conclusion that none of the models is distinctly superior to the others for all cases of predictive orbit generation.

V. Conclusion

The results presented in this paper support the conclusion that, for satellites above 300 km, the Harris-Priester, Jacchia-Roberts, Mass Spectrometer/Incoherent Scatter (MSIS), and Simple Exponential Model (SEM) atmospheric density models all produce essentially comparable orbit determination results when the drag variation parameter is estimated and when orbit quality is measured by overlap comparisons. It is impossible to predict which of the four models will produce the best fit or best predictions for any given orbit determination arc.

From the standpoint of computational efficiency, the use of simpler models (Harris-Priester and SEM) is indicated by the results of this study. Furthermore, use of the SEM for orbit determination when core capacity is limited (e.g., with microprocessor or onboard computers) should provide satisfactory orbit estimation accuracy for most applications.

These results should not be interpreted as a general comparison of atmospheric models; any conclusions about the relative merits of the models must be limited to this highly specialized context—short-arc orbit determination in which an average drag scaling factor is estimated.

Acknowledgments

This work was supported by the Operational Analysis Section, Operational Orbit Support Branch, Goddard Space Flight Center, National Aeronautics and Space Administration, under Contracts NAS 5-24300 and NAS 5-26122. The authors wish to acknowledge the contributions to this study of C.E. Doll, Jr. and J. Teles of the Goddard Space Flight Center and of R.A. Kuseski, formerly of Computer Sciences Corporation.

References

[1]Harris, I. and Priester, W., "Atmospheric Structure and Its Variations in the Region from 120 to 800 km," *COSPAR International Reference Atmosphere (CIRA) 1965,* Space Research IV, North Holland Publishing Co., Amsterdam, The Netherlands, 1965.

[2]Cappellari, J.O., Jr., Velez, C.E., and Fuchs, A.J., eds., "Mathematical Theory of the Goddard Trajectory Determination System," Goddard Space Flight Center Rept. No. X-582-76-77, Greenbelt, Md., April 1976.

[3]Jacchia, L.G., "Revised Static Models of the Thermosphere and Exosphere With Empirical Temperature Profiles," Smithsonian Astrophysical Observatory Special Report No. 332, Cambridge, Mass., May 1971.

[4]Hedin, A.E. et al., "A Global Thermospheric Model Based on Mass Spectrometer and Incoherent Scatter Data: MSIS 1. N_2 Density and Temperature," *Journal of Geophysical Research,* Vol. 82, No. 16, June 1, 1977, pp. 2139-2147.